

# Classification of abelian Schur groups and beyond

Grigory RYABOV

*Abstract.* To study permutation groups containing regular subgroups, Schur introduced the concept of a special subring of the integer group ring, now called an *S-ring* (*Schur ring*) [5]. As Wielandt wrote [7], Schur had conjectured that every S-ring can be constructed from a suitable permutation group containing a regular subgroup. However, as Wielandt showed, this conjecture turned out to be false [6]. In [2], Pöschel introduced the notions of a *schurian* S-ring, i.e., an S-ring that can be constructed from a suitable permutation group containing a regular subgroup, as well as the notion of a *Schur* group, i.e., a finite group such that all S-rings over it are schurian. In the same work, the problem of determining all Schur groups was posed. In general, this problem seems to be hard, in particular because the number of S-rings over a given group can be exponential in the order of the group. The list of all possible abelian Schur groups was obtained by Evdokimov, Kovács, and Ponomarenko [1].

In the talk, we will present a complete classification of abelian Schur groups and discuss some results on nonabelian Schur groups. The talk is partially based on the results from [3, 4].

*About the speaker.* The speaker holds a PhD in mathematics from Sobolev Institute of Mathematics and Novosibirsk State University. He was a postdoctoral research fellow at St. Petersburg Department of V.A. Steklov Institute of Mathematics and Ben-Gurion University of the Negev. Now he is a senior research fellow at Sobolev Institute of Mathematics, an associate professor at Novosibirsk State Technical University, and visiting associate professor at Hebei Normal University.

He has authored or co-authored 31 research papers, some of which have been published in international journals such as *J. Algebra*, *J. Comb. Theory Ser. A*, *J. Algebr. Comb.*, and *Des. Codes Cryptogr.* His primary research interests lie at the intersection of algebra and combinatorics.

## REFERENCES

- [1] S. Evdokimov, I. Kovács, and I. Ponomarenko, *On schurity of finite abelian groups*, *Commun. Algebra*, **44**, No. 1 (2016), 101–117.
- [2] R. Pöschel, *Untersuchungen von s-ringen insbesondere im gruppenring von p-gruppen*, *Math. Nachr.*, **60** (1974), 1–27.
- [3] G. Ryabov, *On schurity of dihedral groups*, *J. Algebra*, **682** (2025), 247–277.
- [4] G. Ryabov, *Classification of abelian Schur groups I*, 40 pp. (<https://arxiv.org/abs/2602.14215>).
- [5] I. Schur, *Zur theorie der einfach transitiven Permutationgruppen*, *S.-B. Preus Akad. Wiss. Phys.-Math. Kl.*, **18**, No. 20 (1933), 598–623.
- [6] H. Wielandt, *Finite permutation groups*, Academic Press, 1964.
- [7] H. Wielandt, *Permutation Groups Through Invariant Relations and Invariant Functions*, Lecture Notes, Department Mathematics Ohio State University, Columbus, 1969.

SOBOLEV INSTITUTE OF MATHEMATICS, NOVOSIBIRSK, RUSSIA  
E-mail: [gric2ryabov@gmail.com](mailto:gric2ryabov@gmail.com)